Int. J. Heat Mass Transfer. Vol. 27, No. 4, pp. 487–495, 1984
Printed in Great Britain

0017-9310/84 \$3.00 + 0.00

DEVELOPMENT OF INVERSE METHODS FOR DETERMINING THERMOPHYSICAL AND RADIATIVE PROPERTIES OF HIGH-TEMPERATURE FIBROUS MATERIALS

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(*Received* 14 *February* 1983 *and in revised form* 16 *May* 1983)

Abstract—The primary objective of this study has been to develop a methodology for determining thermophysical and optical properties of absorbing, emitting and highly scattering fibrous materials and to apply the methodology to a candidate material (zirconia). A model was constructed for predicting the effect of optical (single scattering albedo, extinction coefficient, back scattering fraction, index of refraction) and thermal (conductivity, specific heat) properties on heat transfer across the material. From a sensitivity analysis based on the model, the effect of the radiative properties on the transient thermal behavior was found to be dominated by the scattering albedo, while the effects of thermal conductivity and specific heat were found to be comparable. Nonlinear parameter estimation techniques were used to infer values of the extinction coefficient, back scattering fraction, and thermal conductivity of zirconia from temperature and transmittance measurements made during exposure of samples to solar fluxes of 400 and 600 kW m^{-2}. Using these values with the model, the transmitted solar flux and temperature histories were predicted to within the error bounds of the data.

NOMENCLATURE

b	back scattering fraction
$\bar{b}$	parameter vector
c	specific heat
$E_{b\lambda}$	Planck blackbody function
e	parameter error
F	net radiative flux in material
F^+	forward radiative flux in material
F^-	backward radiative flux in material
F_λ^0	spectral irradiation at front surface
G	total irradiation
h	convection heat transfer coefficient
k	thermal conductivity
L	test sample thickness
n	index of refraction (real part) or number of data points
p	number of properties considered in parameter estimation procedure
S	square root of the sum of the squared deviation between measurements and calculations or square root of the sample variance
S_{r1}	one standard deviation error for the radiation measurements
S_{r2}	one standard deviation error for the radiation parameters fit to the data
S_{t1}	one standard deviation error for the temperature measurements
S_{t2}	one standard deviation error for the thermal parameters fit to the data
T	temperature
t	time
$\bar{X}$	sensitivity coefficient matrix
x	distance in material measured from front face
$\bar{Y}$	vector assigned to storage of measured quantities.

Greek symbols

α	surface absorbtivity
β_i	thermophysical or radiation property
β	best estimate of the property vector, $\bar{\beta}$
β_1, β_2	parameters in the equation for thermal conductivity, $k = \beta_1 + \beta_2(T - T_\infty)$
ε	surface emissivity or error estimate for the measurements
η_i	calculations from the computer model
$\bar{\eta}$	vector assigned to storage of calculated quantities
κ	extension coefficient
ω	single scattering albedo
ρ	reflectivity or density
ρ_0	reflectivity of surface to radiation incident from within the material
λ	wavelength
λ_c	cutoff wavelength for first spectral band
σ	Boltzmann constant or scattering coefficient or population standard deviation
τ	optical thickness, κx
$\bar{\Psi}$	covariance matrix of errors.

Subscripts

b	blackbody
i	index or inside (internal)
o	outside
λ	spectral quantity
∞	ambient condition.

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INTRODUCTION

Semitransparent media have been used in a wide variety of applications, from low temperatures, for materials such as plastics and glass, to high temperatures, for materials such as graphite and thoria. In this study, fibrous materials such as yittria (Y_2O_3), thoria (ThO_2), hafnia (HFO_2), alumina (Al_2O_3) and zirconia (ZO_2) are of particular interest. When such materials are used as insulators on solar parabolic concentrators and central receivers or in high-temperature fuel-fired and electric furnaces, heat transfer through the materials is expected to be by combined conduction and radiation. The materials are semitransparent, strongly scattering, and of high porosity (80–90%). Due largely to the effects of radiation absorption and scattering, heat transfer processes within such materials are poorly understood. A paucity of knowledge concerning fundamental radiation absorption and scattering coefficients of the materials contributes to the uncertainty of heat transfer predictions at elevated temperatures, particularly under conditions for which the materials are exposed to intense radiation from an external source.

This paper is concerned with developing a methodology for determining radiative and thermophysical properties of absorbing, emitting and highly scattering materials subjected to intense solar radiation. The problem falls in a category which is generally referred to as parameter estimation and involves inverting information that cannot be obtained by direct measurements.

Inversion theory has been used to unfold surface temperatures from temperature measurements within the interior of a material [1, 2] and to determine material thermal properties from temperature measurements [3, 4]. Inversion theory has also been applied to emitting–absorbing semitransparent solids [5] and gases [6] to unfold temperature distributions from remote measurements. More recently, the relevance of parameter estimation to numerous problems in science and engineering has been delineated, and specific areas in need of research have been identified [7]. However, in none of the foregoing studies has the theory been applied to materials for which radiation scattering is significant. In contrast, the present study seeks to extend the theory to porous, semitransparent materials for which scattering is important, and to apply more sophisticated statistical inversion techniques to determine parameters important to the radiation and thermal fields.

A methodology is developed for determining thermophysical and radiative properties of fibrous materials used as high-temperature insulators. The specific objectives are: (a) to construct a nonlinear parameter estimation (inversion) model which employs a nonlinear Gauss minimization technique; (b) to describe maximum likelihood estimates that could be used with the technique; (c) to develop procedures for characterizing measurement, computational and parameter errors; and (d) to develop a detailed procedure for using nonlinear parameter estimation techniques with the theoretical model and the measurements. As a specific example, zirconia is used as a fibrous test material, and three types of data are used to determine the specific heat, thermal conductivity, extinction coefficient and backward scattering fraction of the material. The data include temperatures measured within the material, as well as measured incident and transmitted solar fluxes.

MATHEMATICAL MODELS

Inversion methods require a mathematical model to describe the physical processes and an algorithm to determine the parameters. In this section a heat transfer model and a Gauss minimization method for parameter estimation are described.

Heat transfer model

Consideration is given to one-dimensional (1-D), transient heat transfer in a planar layer of a fibrous, high porosity material which is irradiated at its front surface and in which heat transfer is by combined conduction and radiation. The material is assumed to be homogeneous and isotropic. It is also assumed that, for wavelengths smaller than λ_c, the material is semitransparent to radiation; while for wavelengths greater than λ_c, it is opaque. For $\lambda < \lambda_c$ the material absorbs, emits and scatters radiation internally. The density and specific heat of the material are assumed to be independent of temperature, and the thermal conductivity is approximated as a linear function of temperature. Irradiation at the front face of the material is presumed to originate from a high-temperature source and to be uniform and independent of time. Irradiation at the back face is presumed to come from ambient surroundings and is therefore assumed to be negligible.

The energy equation is of the form

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) - \frac{\partial F}{\partial x}, \qquad (1)$$

where the total radiative flux in the semitransparent spectral region is given by

$$F = \int_0^{\lambda_c} (F_\lambda^+ - F_\lambda^-)\, \mathrm{d}\lambda. \qquad (2)$$

The forward and backward spectral radiative fluxes are obtained from a two-flux approximation of the form [8]

$$\frac{\mathrm{d}F_\lambda^+}{\mathrm{d}\tau_\lambda} = -\sqrt{\{3[1-(1-b_\lambda)\omega_\lambda]F_\lambda^+\}} + \sqrt{(3\omega_\lambda b_\lambda F_\lambda^-)} + \sqrt{[3(1-\omega_\lambda)n_\lambda^2 E_{b\lambda}]}, \qquad (3)$$

and

$$-\frac{\mathrm{d}F^-}{\mathrm{d}\tau_\lambda} = -\sqrt{\{3[1-(1-b_\lambda)\omega_\lambda]F_\lambda^-\}} + \sqrt{(3_\lambda \omega_\lambda b_\lambda F_\lambda^+)} + \sqrt{[3(1-\omega_\lambda)n_\lambda^2 E_{b\lambda}]}. \qquad (4)$$

The initial temperature field is assumed to be that of the ambient, $T(x, 0) = T_\infty$, and boundary conditions at the front ($x = 0$) and back ($x = L$) faces are

$$\int_{\lambda_c}^{\infty} (\alpha_\lambda F_\lambda^0 - \varepsilon_\lambda F_{b\lambda})\, d\lambda = -k \frac{\partial T}{\partial x}\bigg|_{x=0} + h[T(0,t) - T_\infty], \quad (5)$$

and

$$-\frac{\partial T}{\partial x}\bigg|_{x=h} = h[T(L,t) - T_\infty] + \int_{\lambda_c}^{\infty} \varepsilon_\lambda E_{b\lambda}\, d\lambda, \quad (6)$$

respectively. Appropriate boundary conditions for the forward and backward radiative fluxes are obtained from radiative balances at the front ($x = 0$) and back ($x = L$) faces and are given by

$$F_\lambda^+(0) = (1 - \rho_{o\lambda})F_\lambda^0 + \rho_{i\lambda} F_\lambda^-(0), \quad (7)$$

and

$$F_\lambda^-(L) = \rho_{i\lambda} F_\lambda^+(L), \quad (8)$$

respectively.

Parameter estimation

Working with the foregoing model and data taken for zirconia samples, selected optical properties (ω and b) and the thermophysical properties (c and k) were determined by using the following procedure.

Data, such as material temperatures, heat fluxes, and reflectance, are stored in a vector $\bar{Y}$, while corresponding results obtained by solving the model equations are stored in a vector $\bar{\eta}$. Knowing $\bar{Y}$ and $\bar{\eta}$, the desired properties, which are designated as β_i and stored in a vector $\bar{\beta}$, may be estimated by using Gauss minimization. The resulting expression for the best estimate, β, of a property is of the form [8, 9]

$$\beta = \bar{b} + \bar{p}^{-1}\{\bar{X}^T \bar{\Psi}^{-1}\{\bar{Y} - \bar{\eta}(\bar{b})\}\}, \quad (9)$$

where

$$\bar{p} = \bar{X}^T \bar{\Psi}^{-1} \bar{X}. \quad (10)$$

A first estimate of the property is given by the vector $\bar{b}$, after which the data, $\bar{Y}$, and the calculations, $\bar{\eta}(\bar{b})$, are used to determine $\bar{\beta}$ for the first iteration. For the next iteration, $\bar{b}$ is set equal to $\bar{\beta}$ from the first iteration and a new $\bar{\beta}$ is found. The process is continued until the value for $\bar{\beta}$ converges to within the desired accuracy. The matrix $\bar{X}$ is the sensitivity matrix [10] for the parameters

$$\bar{X} = \begin{bmatrix} \frac{\partial \eta_1}{\partial \beta_1} & \frac{\partial \eta_1}{\partial \beta_2} & \cdots & \frac{\partial \eta_1}{\partial \beta_p} \\ \cdot & \cdot & & \cdot \\ \frac{\partial \eta_n}{\partial \beta_1} & \frac{\partial \eta_n}{\partial \beta_2} & \cdots & \frac{\partial \eta_n}{\partial \beta_p} \end{bmatrix}, \quad (11)$$

where there are p properties and n computations or data points. For example, in applying this method to determine the thermophysical properties (c and k), use was made of temperature measurements at four different locations in the material, in which case $p = 2$ and $n = 4$. The sensitivity matrix provides a measure of the effect of property changes on model predictions.

The error matrix appearing in equations (9) and (10) is expressed as

$$\bar{\Psi} = \mathrm{cov}(\bar{\varepsilon}) \equiv \begin{bmatrix} V(\varepsilon_1) & \mathrm{cov}(\varepsilon_1, \varepsilon_2) & \cdots & \mathrm{cov}(\varepsilon_1, \varepsilon_n) \\ \mathrm{cov}(\varepsilon_1, \varepsilon_1) & V(\varepsilon_2) & \cdots & \mathrm{cov}(\varepsilon_2, \varepsilon_n) \\ \cdot & \cdot & & \cdot \\ \mathrm{cov}(\varepsilon_1, \varepsilon_n) & \mathrm{cov}(\varepsilon_2, \varepsilon_n) & \cdots & V(\varepsilon_n) \end{bmatrix}, \quad (12)$$

where ε_i is the error associated with each datum point. The covariance is defined as

$$\mathrm{cov}(\varepsilon_i, \varepsilon_j) \equiv E(\varepsilon_i \varepsilon_j) - \{E(\varepsilon_i)\}\{E(\varepsilon_j)\}, \quad (13)$$

and the variance as

$$V(\varepsilon_i) \equiv E\{[\varepsilon_i - E(\varepsilon_i)]^2\} = \sigma_i^2. \quad (14)$$

Equation (12) includes the possibility that the data can be correlated.

The covariance matrix for the property estimate β is

$$\mathrm{cov}(\beta) = (\bar{X}^T \bar{\Psi}^{-1} \bar{X})^{-1}, \quad (15)$$

where the diagonal components of equation (15) are the variances for the properties. An estimate of the one standard deviation error for the properties can be obtained from equation (15). Potential complications associated with using the Gauss minimization method, such as uniqueness, convergence and the adequacy of various statistical assumptions, are discussed elsewhere [9, 10].

EXPERIMENTAL PROCEDURES

The experiments were designed to obtain temperature and transmitted radiation measurements for zirconia samples subjected to intense solar radiation, and the measurements were used to evaluate parameters in the computer model. Temperatures were expected to vary from 400 to 2000 K. All of the experiments were performed at the New Mexico State University Solar Furnace [11], which could provide for sample irradiation up to 700 kW m^{-2}. The irradiation is diffuse to within 5% over a cone angle of 80°. It should be noted that all radiance measurements of this study are of a nonspectral (total) nature. Hence it is only possible to determine the spectrum averaged (from $\lambda = 0$ to λ_c) radiative properties of the material. In principle, if spectral data were available, it would be possible to determine spectral radiative properties using the above model.

Two groups of tests were performed, one involving temperature measurements and the other transmission measurements. Temperature measurements were made at five distances from the irradiated surface of zirconia samples, using 30 gauge, chromel–alumel and 26 gauge,

Table 1. Test matrix for temperature measurements

Depth of thermocouple placement (mm)	Irradiation (kW m^{-2})	Number of runs
1.6	400	4
1.6	600	4
3.8	400	4
3.8	600	4
6.4	400	4
6.4	600	4
8.9	400	4
8.9	600	4
11.1	400	4
11.1	600	4

platinum–platinum 10% rhodium thermocouples (Table 1). The samples were 12.7 mm thick. To minimize disruption of the temperature field by the thermocouples, separate samples were used for each temperature measurement. The radiation transmission measurements were made for different sample thicknesses, using circular foil heat flux gauges (Table 2).

Results of the radiation transmission measurements are shown in Table 3. Circular foil heat flux gauges were used to measure the radiant heat flux incident on the zirconia and the transmitted flux. To insure that no emission from the back surface was included in the measurement, a fast shutter was employed. With the shutter closed, there was no energy incident on the sample. After setting the irradiation, the shutter was opened, the sample was exposed to the incident radiation, and a reading of the transmitted heat flux was quickly taken. With the shutter open for less than 2 s, the back surface temperature was kept at or near ambient conditions. Several readings were taken for each test sample at each of two heat flux settings and were repeated on different days. In this way, the effect of spurious errors was reduced.

The estimated standard deviation, S_{r1}, of the radiation measurement is

$$S_{r1} = \left[\frac{1}{(n-1)}\sum_{i=1}^{n}(Y_i - \bar{Y})^2\right]^{1/2}, \tag{16}$$

where Y_i is the datum point, $\bar{Y}$ is the mean of the data, and $(n-1)$ is the number of data points, n, minus the constraints, 1. There is one constraint because S_{r1} is found from a sample of data, rather than from the entire population [12].

Table 2. Test matrix for radiation transmission measurements

Sample thickness (mm)	Irradiation (kW m^{-2})	Number of runs
3.2	400	4
3.2	600	4
6.4	400	4
6.4	600	4
9.5	400	4
9.5	600	4

Results of the temperature measurements are shown in Figs. 1 and 2 for irradiations of 400 and 600 kW m^{-2}, respectively. A major concern of these measurements is the error introduced by the presence of the thermocouple. Since the thermocouple has a thermal conductivity that is considerably larger than that of the zirconia, the temperature of the material is influenced by its presence. A computer program was therefore written to estimate the effect of the thermocouple in the zirconia, and the results were used to correct the thermocouple measurements. Discussion of the thermocouple correction analysis, basic statistical assumptions, and measurement errors is given by Matthews [9]. Test design considerations have shown that, the optimal range for taking data is between 60–260 s. This range was determined by applying a detailed sensitivity analysis to the problem [9].

Sensitivity analysis

A sensitivity analysis was performed to determine the influence of the various properties on thermal and radiative conditions. The pertinent radiative properties include the single scattering albedo, ω, the back scattering fraction, b, the extinction coefficient, κ, the real part of the refractive index, n, and the surface reflectivity, ρ_0. Four data points were used to estimate the relative influence of these properties. Three of the points involved the transmission measurements of this study (for three different sample thicknesses), and the fourth point involved zirconia surface reflectance data [13]. Results of the sensitivity analysis are summarized in Table 4, where modified sensitivity coefficients obtained for prescribed conditions are tabulated for the transmittance and reflectance. From the results, it is clear that a change in the single scattering albedo, ω, causes a large change to occur in the transmitted and reflected radiation fluxes. This behavior is attributed to the large value (approximately 0.99) of ω associated with zirconia [14, 15]. A slight change in ω therefore

Table 3. Transmittance measurements as a function of sample thickness

Thickness (mm)	3.2		6.4		9.5	
Input heat flux (kW m^{-2})	400	600	400	600	400	600
Transmitted heat flux (mean of measurements) (kW m^{-2})	9.2	13.2	1.2	1.7	0.6	0.93
Estimated standard deviation, S_{r1} (kW m^{-2})	0.5	1.2	0.1	0.16	0.05	0.07
Transmitted flux/incident flux (%)	2.4	2.2	0.3	0.28	0.15	0.16

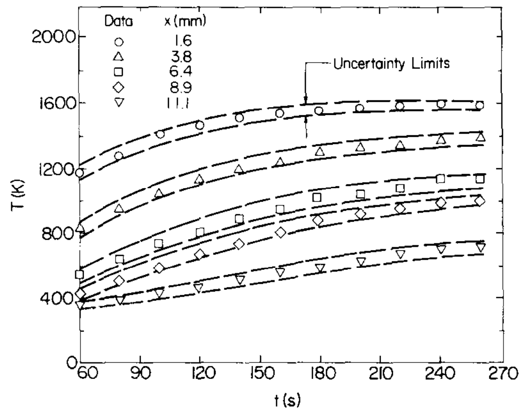

FIG. 1. Temperature measurements and uncertainty limits ($G = 400$ kW m^{-2}).

provides for a large change in the transmittance and reflectance. By comparison, the effects of changes in the other properties are negligible.

Results of a sensitivity analysis performed for the thermal properties c and k are shown in Table 5. Calculations were performed at the four thermocouple locations as a function of time. The modified sensitivity coefficient, $\beta(\partial\eta/\partial\beta)$, represents a change in the calculated temperature with respect to a change in the property, multiplied by the property. A negative coefficient implies that the temperature decreases with an increase in the property. For each case an increase in the specific heat decreases the temperature. In contrast, an increase in the thermal conductivity decreases the temperature near the front of the sample and increases the temperature toward the back. The effects of the two properties are comparable. A similar analysis for the heat transfer coefficient, h, showed that conditions are not as sensitive to changes in h as to c and k.

In summary, the sensitivity analysis for the radiation parameters reveals that the effect of the single scattering albedo, ω, is dominant. A small error in its value can cause significant errors in calculating the transmittance and reflectance of the material. Of the other radiation parameters (back scattering fraction, extinction coefficient and real part of the index of refraction),

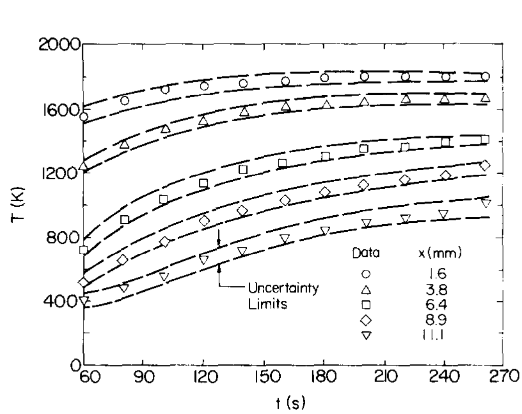

FIG. 2. Temperature measurements and uncertainty limits ($G = 600$ kW m^{-2}).

Table 4. Modified sensitivity coefficient, $\beta(\partial\eta/\partial\beta)$, for the radiation properties (computed for $G = 400$ kW m^{-2}, $\omega = 0.99$, $b = 0.2506$, $\kappa = 8969$ m^{-1} and $n = 1.6$)

	Transmittance Sample thickness 3.2 mm	6.4 mm	9.5 mm	Reflectance
ω	192.8	37.7	9.39	15.09
b	−1.354	−0.0737	0.0631	2.784
K	−2.979	−0.1891	−0.0207	0.039
n	0.882	0.1239	0.0949	−0.113
ρ_0	0.042	−0.0023	0.0006	0.005

errors in the back scattering fraction would have the most significant effect on the reflectance calculations, while errors in the extinction coefficient have a more significant influence on the transmittance calculations. For the thermal properties, the sensitivity analysis reveals that the thermal conductivity and the specific heat are of equal importance for the temperature calculations.

On the basis of the foregoing results, the decision was made to use the methods of nonlinear parameter estimation to determine the back scattering fraction and the extinction coefficient, with values of the scattering albedo and the real part of the index of refraction obtained from refs. [14–16] ($\omega = 0.99$, $n = 1.6$). In addition the total hemispherical reflectance was assigned a value of $\rho_0 = 0.65$ [13].

Property determination

Using four datum points ($n = 4$), corresponding to three transmittance measurements for $G = 400$ kW m^{-2} and the total hemispherical reflectance, and the prescribed values of ω and n, the nonlinear parameter estimation technique was used to determine the values of b and κ. Beginning with assumed values of $b = 0.25$ and $\kappa = 8500$ m^{-1}, iterative calculations were performed and a comparison of the calculated and measured transmittance and reflectance values is shown in Table 6. The standard deviation

$$S_{r2} = \left[\frac{1}{n-1}\sum_{i=1}^{n}[(Y_i - \bar{Y})^2]\right]^{1/2}, \tag{17}$$

decreases with each iteration and the values of b and κ agree to within 1% from the second to the third iteration. A good initial guess was instrumental in keeping the number of iterations small. A poor guess could lead to many more iterations before convergence is achieved or could preclude the occurrence of convergence. Using equation (15), the covariance matrix of the properties is found to be

$$\begin{bmatrix} 0.001063 & 0 \\ 0 & 304\,920 \end{bmatrix}, \tag{18}$$

from which the one standard deviation error for the properties is

$$e_b = 0.0326, \tag{19}$$

$$e_\kappa = 552\ (\mathrm{m}^{-1}). \tag{20}$$

The sensitivity analysis performed for the radiation properties showed the effect which changes in ω and n can have on estimates of the transmittance and reflectance. Table 4 revealed a far greater dependence on ω than on n. In Table 7 the effect of variations in ω and n on values of b and κ determined from the nonlinear parameter estimation is shown. It is evident that the results obtained for b and κ are very sensitive to a small change in ω. In contrast, a change in n has little effect on the calculations, and the corresponding changes in b and κ are well within the one standard deviation error band for the parameters. Property values for which S_{r2} is minimized are $\omega = 0.99$, $n = 1.6$, $b = 0.2506 \pm 0.0326$, and $\kappa = 8969 \pm 552$ (m^{-1}). In Table 8 predictions based on these property values are compared with measured results for input heat fluxes of 400 and 600 kW m^{-2}. The calculations are within the error band of the data except for the transmittance of the 6.4 mm sample at 600 kW m^{-2}. The greatest transmitted flux is for the 3.2 mm test sample, and the calculations fit the data very well for this thickness (the deviation is less than 2% for the transmitted flux and 1% for the reflectance data).

Nonlinear parameter estimation methods were also used to determine the specific heat and thermal conductivity, with the conductivity assumed to be a linear function of temperature [17]. Results are shown in Table 9. Although each set of properties was used to predict the measured temperature histories, the best overall agreement was obtained from the use of average property values

$$c = 1750 \pm 50 \text{ J kg}^{-1}\text{ K}^{-1},$$

$$k = 0.07 + 0.933 \times 10^{-4}(T - T_\infty) \pm 0.014 \text{ W m}^{-1}\text{ K}^{-1}.$$

Table 5. Modified sensitivity coefficient, $\beta(\partial\eta/\partial\beta)$, for the thermal properties (computed for $G = 400$ kW m^{-2}, $c = 1750$ J kg^{-1} K^{-1}, $k = 0.001693$ W m^{-1} K^{-1})

Sample thickness (mm)	1.6		6.4		8.9		11.1	
Property time (s)	c	k	c	k	c	k	c	k
60	−214.2	−97.5	−404.1	150.9	−165.5	95.8	−57.1	39.8
90	−145.4	−85.2	−506.7	127.6	−347.2	168.9	−191.9	122.1
120	−115.2	−77.0	−499.0	83.9	−480.2	199.2	−347.7	202.4
150	−99.5	−71.0	−458.6	53.0	−551.1	198.6	−471.4	251.1
180	−90.4	−65.3	−415.8	36.3	−567.4	180.0	−521.9	255.7
210	−84.5	−60.4	−373.5	26.0	−535.2	148.2	−511.1	228.9
240	−77.9	−57.2	−326.0	14.5	−468.8	110.2	−461.7	191.8

Table 6. Parameter estimation results for b and k ($\omega = 0.99$, $\eta = 1.6$, $G = 400$ kW m^{-2})

Iteration	b	κ (m^{-1})	Transmitted heat flux (kW m^{-2}) (3.2 mm)		(6.4 mm)		(9.5 mm)		Reflectance		S_{r2}
			Cal.	Data	Cal.	Data	Cal.	Data	Cal.	Data	
1	0.2477	8950	9.43	9.2	1.08	1.2	0.80	0.6	0.643	0.65	0.232
2	0.2505	8967	9.26	9.2	1.08	1.2	0.80	0.6	0.645	0.65	0.170
3	0.2506	8969	9.23	9.2	1.08	1.2	0.80	0.6	0.645	0.65	0.166

Table 7. Effect of changes in ω and n on values of b and κ determined from the nonlinear parameter estimation method

	Data	Case I	II	III
ω		0.99	0.99	0.98
n		1.60	1.68	1.60
Transmitted flux, 3.2 mm (kW m^{-2})	9.2 ± 0.5	9.23	9.15	9.24
Transmitted flux, 6.4 mm (kW m^{-2})	1.2 ± 0.1	1.08	1.11	0.36
Transmitted flux, 9.5 mm (kW m^{-2})	0.6 ± 0.05	0.80	0.85	0.06
Reflectance	0.65	0.645	0.642	0.64
b		0.2506	0.2852	0.438
κ (m^{-1})		8969	8737	4616
S_{r2}		0.017	0.019	0.0707
Number of iterations used in estimation technique		3	3	5

Table 8. Comparison of measured transmitted radiation flux and reflectance with predictions for $\omega = 0.99, b = 0.2506, \kappa = 8969$ m^{-1} and $n = 1.6$

Input heat flux ($kW\ m^{-2}$)	Transmitted heat flux ($kW\ m^{-2}$) 3.2 mm		6.4 mm		9.55 mm		Reflectance		S_{r2}
	Cal.	Data	Cal.	Data	Cal.	Data	Cal.	Data	
400	9.23	9.2±0.5	1.08	1.2±0.13	0.8	0.6±0.05	0.645	0.65	0.166
600	13.4	13.2±1.5	1.30	1.7±0.16	0.86	0.93±0.07	0.645	0.65	0.320

Table 9. Results of thermophysical properties determined from nonlinear parameter estimation

Case	Input heat flux ($kW\ m^{-2}$)	Specific heat ($J\ kg^{-1}\ K^{-1}$)	Thermal conductivity ($W\ m^{-1}\ K^{-1}$)	Number of iterations
I	400	1892±60.0	$[0.07+0.1137\times10^{-3}(T-T_\infty)]\pm0.018$	3
II	600	1608±40.0	$[0.07+0.7289\times10^{-4}(T-T_\infty)]\pm0.012$	5

The comparison is shown in Figs. 3 and 4, and standard deviation errors are summarized in Table 10. The errors are less than 10% of the average temperature for every thermocouple reading, except the third.

Table 10. One standard deviation for average temperatures

Input heat flux ($kW\ m^{-2}$)	Depth (mm)	Average temperature (K)	S_{t2} (K)
400	overall	976	±93.20
	1.6	1481	±95.59
	3.8	1188	±108.51
	6.4	901	±116.51
	8.9	765	±55.88
	11.1	547	±70.66
600	overall	1262	±93.97
	1.6	1753	±136.49
	3.8	1561	±92.03
	6.4	1207	±34.02
	8.9	979	±75.35
	11.1	758	±96.83

CONCLUSIONS

Methodology has been developed and tested to determine thermophysical and radiative properties of a semitransparent, fibrous material from indirect measurements. As a specific example zirconia was used as a test material. The method of nonlinear parameter estimation was used with experimental data to determine the unknown parameters in an optimal fashion. The parameters found in this manner should be considered as average values over the solar spectrum and are limited in accuracy by the numerical algorithm used to solve the model equations and the accuracy of the data. The radiation parameters included the extinction coefficient, the single scattering albedo, the back scattering fraction and the real part of the complex index of refraction. Because of limitations in the testing, the single scattering albedo was not found in an optimal fashion. Although the value of 0.99 used for the single scattering albedo is believed to be within approximately ±0.005 of the true value, use of the

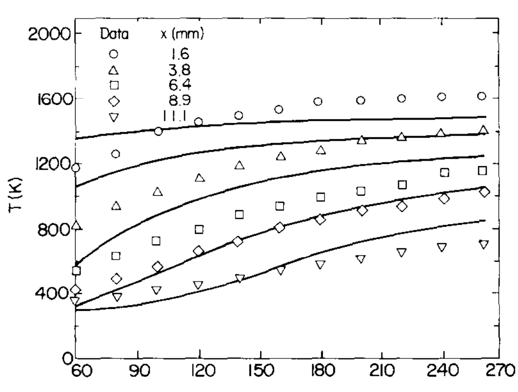

FIG. 3. Comparison of measured and calculated (solid line) temperatures for $G = 400\ kW\ m^{-2}$.

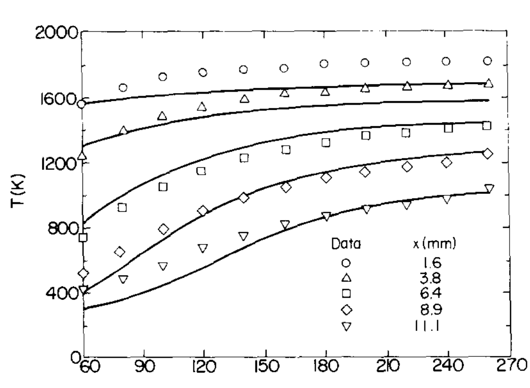

FIG. 4. Comparison of measured and calculated (solid line) temperatures for $G = 600\ kW\ m^{-2}$.

estimation technique is extremely sensitive to this value, and results obtained for the back scattering fraction and the extinction coefficient must be viewed as first estimates.

Values of the specific heat and temperature dependent thermal conductivity determined from the estimation method are in the range of data reported in the literature. The predicted temperature distributions, using the estimated radiation and thermal parameters, agree within 10% in most cases with the measured temperatures. The discrepancy between results is attributed primarily to uncertainties in the measurements, particularly near the front face of the test sample, and the assumption that the specific heat of the test materials is independent of temperature.

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DEVELOPPEMENT DES METHODES INVERSES POUR DETERMINER LES PROPRIETES THERMOPHYSIQUES ET RADIATIVES DES MATERIAUX FIBREUX A HAUTE TEMPERATURE

Résumé—Le premier objectif de l'étude est de développer une méthologie pour déterminer les propriétés thermophysiques et optiques des matériaux fibreux absorbants, émetteurs et fortement diffusants et d'appliquer cette méthodologie à un matériau particulier (zircone). Un modèle est construit pour calculer les propriétés optiques (albedo simplement diffusant, coefficient d'extinction, fraction de rétrodiffusion, indice de réfraction) et thermiques (conductivité, capacité thermique). L'effet des propriétés radiatives sur le comportement thermique est dominé par l'albédo tandis que les effets de conductivité thermique et de capacité thermique sont comparables. Les techniques d'estimation de paramètre non-linéaire sont utilisées pour inférer les valeurs du coefficient d'extinction, de la fraction de rétrodiffusion, de la conductivité thermique à partir des mesures de température et de transmittance faites pendant l'exposition d'éprouvette de zircone à des flux solaires de 400 et 600 kW m^{-2}. A partir de ces valeurs mises dans le modèle, le flux solaire transmis et l'histoire des températures sont prédites dans les limites de précision des mesures.

ENTWICKLUNG VON INVERSEN METHODEN ZUR BESTIMMUNG DER THERMOPHYSIKALISCHEN UND STRAHLUNGSEIGENSCHAFTEN VON FASERIGEN MATERIALIEN BEI HOHER TEMPERATUR

Zusammenfassung—Das Ziel dieser Studie war die Entwicklung einer Methode zur Bestimmung der thermophysikalischen und optischen Eigenschaften von absorbierenden, emittierenden und stark streuenden faserigen Materialien und die Anwendung der Methode auf einen ausgewählten Stoff (Zirkonium). Für die Bestimmung des Einflusses der optischen Eigenschaften (einfach streuendes Albedo, Extinktionskoeffizient, Rückstreuungsanteil, Brechungsindex) und der thermischen Eigenschaften (Wärmeleitfähigkeit, spezifische Wärme) auf den Wärmetransport durch das Material wurde ein Modell entwickelt. Eine Empfindlichkeitsanalyse des Problems mit dem Modell ergab, daß bezüglich des Einflusses auf das Übergangsverhalten bei den Strahlungseigenschaften das streuende Albedo vorherrschend war; dagegen waren die Einflüsse der Wärmeleitfähigkeit und der spezifischen Wärme vergleichbar. Um auf Werte des Extinktionskoeffizienten, der Rückstreuung und der Wärmeleitfähigkeit von Zirkonium schließen zu können, wurde eine Technik der nichtlinearen Parameterabschätzung verwendet, wobei Temperatur und Transmissionsmessungen bei der Bestrahlung von Proben mit Intensitäten von 400 und 600 kW m^{-2} durchgeführt wurden. Bei Anwendung des Modells auf diese Werte konnten der übertragende Strahlungsfluß und die zeitlichen Temperaturverläufe innerhalb der Fehlergrenzen der Daten berechnet werden.

РАЗРАБОТКА ОБРАТНЫХ МЕТОДОВ ОПРЕДЕЛЕНИЯ ТЕПЛОФИЗИЧЕСКИХ И ЛУЧИСТЫХ ХАРАКТЕРИСТИК ТЕРМОУСТОЙЧИВЫХ ВОЛОКНИСТЫХ МАТЕРИАЛОВ

Аннотация—Исследование проведено с целью разработки методов определения теплофизических и оптических характеристик поглощающих, излучающих и сильно рассеивающих волокнистых материалов и их применения к специально выбранному материалу – цирконию. Построена модель для расчета влияния оптических (единичное альбедо рассеяния, коэффициент затухания, доля отражения назад, индекс преломления) и тепловых (теплопроводность, удельная теплоемкость) свойств на теплоперенос в поперечном сечении материала. На основе изучения модели показано, что влияние лучистых свойств на неустановившийся тепловой режим зависит от альбедо рассеяния, в то время как эффекты теплопроводности и теплоемкости сравнимы по величине. Методы оценки нелинейного параметра использовались для определения значений коэффициента затухания, вклада рассеяния назад и теплопроводности циркония на основе значений температуры и коэффициента пропускания, измеренных при облучении образцов потоками солнечной энергии мощностью 400 и 600 квт $м^{-2}$. Используя полученные значения в модели, рассчитаны изменения во времени потока солнечной энергии и температуры в пределах экспериментальной погрешности.